DESIGN, FABRICATION AND TEST OF A TORSIONAL-VIBRATION INDUCER

BY

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PREFACE

The work on this project was carried out during the period between February 1951 and May 1951 at the United States Naval Postgraduate School.

The original idea for the project was suggested by Dr. E. K. Gatcombe, Associate Professor of Mechanical Engineering at the Naval Postgraduate School. Dr. Gatcombe pointed out the long felt need for a unit which would induce torsional vibrations in a rotating shaft, and further related some of his own personal experiences with the design problems associated with such a unit. It was pointed out that if such a device could be perfected, the advantages to design engineers and to the manufacturers of rotating machinery would be many and far reaching.

The author wishes to express his appreciation to the many persons who have contributed their time, skills and knowledge toward making this project a success. The personnel of the U. S. Naval Engineering Experiment Station were responsible for the original fabrication of the devices herein described. The personnel of the machine shop at the U. S. Naval Postgraduate School provided their willing and able assistance in making the many alterations which were subsequently found to be necessary or desirable. The Postgraduate School staff Photographer also gave unstintingly of his time and skill in reproducing the many oscillograms made in the process of this project and the illustrations which appear in this paper.

Appreciation is especially due to Dr. E. K. Gatcombe for his constant interest and invaluable guidance both in the original design and operation of the equipment and in the preparation of this paper.

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TABLE OF SYMBOLS

C	a constant
T	period of torsional vibration (sec.)
ω_n	natural frequency of torsional vibration (cycles per sec.)
I	centroidal moment of inertia, general (slug-in^2)
k	spring constant of shaft in torsion, general (lb. in. per radian)
l	length of test shaft (in.)
G	modulus of rigidity of test shaft
I_p	polar moment of inertia of test shaft (in.^{4})
I_1	centroidal moment of inertia of pick-up gear.
l_1	length of equivalent shaft between pick-up gear and nodal point of shaft.
I_2	centroidal moment of inertia of driving rotor (slug-in^2).
l_2	length of equivalent shaft between rotor and nodal point of shaft.
f	frequency of torsional vibration (cycles per sec.)
l_{eq}	equivalent length of shaft (inches)
d	actual diameter of shaft (inches)
d_{eq}	diameter of equivalent shaft (inches)
M	mass (slugs)
R	outside radius of hollow cylinder
r	inside radius of hollow cylinder; radius of solid cylinder
a	height of rotor blade (inches)

b thickness of rotor blade (inches)

r_c centroidal radius of blades (inches)

θ angular deflection of test shaft (radians)

CHAPTER I

INTRODUCTION

The designers and manufacturers of rotating machinery have long felt an urgent need for a practical device which will induce torsional vibrations in rotating shafts. In order to provide all the information necessary for designers to understand torsional vibration, its cause and effect, and methods for eliminating or controlling the vibration, a torsional vibration inducer must be capable of maintaining the specimen under test in a state of rotation with an oscillating torque superimposed upon the rotation.

The usual method for studying the effects of torsional vibration on a shaft consists of setting up a test specimen, which is equivalent to the shaft under study, with one end rigidly clamped while the free end is oscillated by means of some torque producing device. In general it is assumed that the rotative effect of the shaft may be disregarded and the results of a fixed end test may be taken as indicative of the effects of torsional vibration on a rotating shaft. This, however, is not always true. In one particular case the Scintilla Magneto Company experienced magneto shaft failures wherein data obtained by means of a fixed-end test could not be correlated with actual conditions.

The device used for the project described in this thesis to produce the oscillating torque consists of an air driven motor in which provision has been made to introduce two sets of auxiliary air jets, acting on auxilliary rotor blades, in

such a way as to produce alternating boosting and bucking impulses on the rotor of the driving motor. The resultant oscillating torque is transmitted through a test specimen of reduced cross-section to an electrical pick-up device.

The pick-up device consists of a spur gear mounted on a shaft which is rotated by the test specimen, and a horseshoe magnet which is provided with necessary exciting and pick-up coils. The poles of the magnet span one-tooth space and the leads from the pick-up coil are attached to one channel of a multi-element, string type oscillograph. As the teeth of the spur gear rotate past the poles of the magnet, the periodic variation of the reluctance of the magnetic path causes a signal to be generated which is picked up by the pick-up coil and gives a characteristic wave form on the oscillograph.

The variation of the angular velocity of the gear caused by the oscillating torque of the driving motor should result in a frequency modulated wave, the characteristics of which would be indicative of the angular acceleration and deceleration and should furnish the means for analysis of the induced torsional vibration in the specimen.

CHAPTER II

DESIGN OF TORSIONAL-VIBRATION INDUCER

The basic idea of using compressed air as a driving medium and also to produce the oscillating torque in the driving motor was suggested from observation of the rapid reaction of air turbines such as those used to drive gyro-mechanisms in torpedoes and other such gear. It was observed that a torpedo gyro can attain a speed of several thousand revolutions per minute in a matter of seconds. It was therefore reasoned that, due to the almost instantaneous reaction produced by a jet of compressed air, such a jet applied in a proper manner to an air rotor should be capable of producing the oscillating torque effect desired in this device.

Since it was the purpose of this project first to produce a unit which would induce torsional vibrations in a shaft and then to analyze the experimental results, no attempt was made to design a driving unit which would produce a known driving torque or oscillating torque. Instead, the design of the driving unit was carried out entirely empirically, with many of the details and dimensions being dictated by size and shape of existing equipment which was modified to suit the project at hand.

For purposes of simplifying fabrication it was decided to use the frame of a 1 HP electric motor as a frame for the air driven motor and vibration inducer unit. The bearings and the outer shell of the motor frame were useable, while a new liner and a rotor fitted with suitable vanes and mounted on a shaft fitted to the motor bearings were fabricated and installed.

Through the outer shell and the liner were installed two driving nozzles of 1/2" O.D. copper tubing, two boosting nozzles of 3/8" O.D. copper tubing, and two bucking nozzles of 3/8" O.D. copper tubing. The driving nozzles were placed diametrically opposite and on the circumferential center line of the motor frame. The boosting nozzles, so arranged as to assist the driving nozzles were placed diametrically opposite but at opposite ends of the motor frame. The bucking nozzles, so arranged as to oppose the driving nozzles were also placed diametrically opposite and are also at opposite ends of the frame. All nozzles enter the liner normal to the axis of the motor frame and make an angle of 30° with the tangent of their point of entry. All nozzles are supplied from a common manifold which has three separate control valves, one each for the driving, boosting, and bucking nozzles respectively.

The driving rotor consists of a steel drum with twenty four removable brass blades, which are installed in slots and secured by end locking rings. The blades are so shaped as to present their surface over two-thirds of the length of the rotor drum, at either end, as desired. For any arrangement of blades, the driving nozzles will impinge on the blades throughout the circumference. However, the blades may be arranged in various manners such that the boosting and bucking nozzles will impinge on blades alternately through various fractions of the circumference.

For the purposes of the investigation herein under discussion, the blades were arranged so as to provide a boosting impulse during half a revolution and a bucking impulse during

the other half revolution. The air manifold was supplied with compressed air ranging in pressure from 50 to 90 pounds per square inch.

The specimen under test consisted of a reduced area shaft, connected to the driving shaft at one end and the pick-up gear shaft at the other end, both connections being made by means of sleeve couplings secured by taper pins. Various sizes of reduced sections were tried in an effort to reduce the natural frequency of the vibrating system to a safe and convenient rotating speed. The size which was finally adopted was a 0.3 inch diameter, 10 inches long. The theoretical development for the natural frequency of the system appears in Appendix I.

It had originally been intended to reduce the moment of inertia of the pick-up gear by reducing its rim thickness to 1/2 inch, but machining difficulties necessitated the abandonment of this project. This resulted in a theoretical nodal point in the test shaft located only about 2 inches from the gear end of the test shaft. At a later stage of the test a flywheel was added at the driving end which resulted in moving the theoretical nodal point to a distance of 6.2 inches from the gear end of the test shaft.

CHAPTER III

MEASURING INSTRUMENTS

Again to facilitate and expedite fabrication, the major element of the pick-up unit, a 46 tooth spur gear was obtained from a diesel engine which was being disposed of by the Engineering Experiment Station. This gear was provided with a suitable hub and shaft and the shaft was fitted with ball bearings and mounted on channel iron pedestals.

The pick-up magnet was fabricated from a 5" diameter soft steel billet. It consists of a circular shaped core of rectangular cross section with poles so arranged to span one tooth interval of the spur gear. Two identical coils were wound, one on each side of the core, and each consisting of 1000 turns of No. 27 varnished copper magnet wire. One of these coils was intended for use as the pick-up coil while the second was intended to provide an exciting current for the pick-up magnet. It was subsequently observed that after the magnet had been flashed, normally, no external excitation was required. The terminals of the pick-up coil were connected to the input terminals of one element of the oscillograph while a 12-volt battery was connected across the exciting coil to provide exciting current if and when necessary.

Several attempts were first made to use a cathode ray oscilloscope to reproduce the wave form generated by the pick-up coil. However, this was found to be impracticable due to the fact that it was impossible to devise a means of holding a standing wave on the oscilloscope. Other drawbacks with the oscilloscope were first, the incedence of many extraneous sig-

nals caused by noise and stray frequencies and also the inability to introduce a timing wave into the single element cathode ray oscilloscope.

The cathode ray oscilloscope was therefore discarded in favor of the Westinghouse Type PA Multi-Element String Type Oscillograph. This device provided the additional channel for the timing wave as well as an attached camera for the simultaneous recording of both the signal wave and the timing wave on sensitized paper or film. Later a General Electric, type PM-10 oscillograph was used as the recording instrument. Due to its higher film speed the GE instrument provided a more easily analyzed wave form.

CHAPTER IV

TEST PROCEDURE AND RESULTS

In its original form, the inducer unit was assembled and run without the flywheel on the driving end and with a mild steel test shaft having a reduced cross section of 5/8" dia. This arrangement was found to have a theoretical natural frequency of torsional vibration in the vicinity of 1500 revolutions per minute. After a check of the running characteristics of the unit, it was decided, in the interests of safety, to limit the speed to 1000 revolutions per minute. However, the unit was operated at speeds below 1000 revolutions per minute in a preliminary attempt to determine the characteristics of the wave forms to be expected from the pick-up gear magnet.

A cathode ray oscilloscope was first used as the indicating unit, but difficulty was experienced in obtaining a standing wave. The indications were that there were other signals being picked up and projected onto the oscilloscope screen. All leads were carefully shielded and the shielding was grounded, but the hash persisted, and no satisfactory wave form could be recognized. It was therefore decided to use a string type recording oscilloscope.

One element of a Westinghouse multi-element string type oscillograph was connected to the output from the pick-up gear magnet, and a signal from a Hewlitt-Packard audio-oscillator was introduced into a second channel of the oscillograph. Again it was not possible to obtain a satisfactory wave form on the viewing screen of the oscillograph, so a photographic record was made. The resulting wave form was found to be a

wave of the expected sinusoidal form and having the frequency generated by the rotating gear teeth (46x revolutions per sec.) superimposed upon a second wave having a frequency equal to 1/23 of the pick-up wave. This latter wave thus had a frequency of 2 cycles per revolution of the pick-up gear. Figure I illustrates the form of this wave. Later investigation led to the conclusion that the lower frequency wave was caused in some manner by residual magnetism in the pick-up gear. The gear was found to have a four-pole arrangement, with two north poles diametrically opposite each other and two south poles also diametrically opposite each other and spaced 90 degrees from the north poles on the rim of the gear.

With information available concerning the expected form of the wave, the test shaft was redesigned to reduce the theoretical natural frequency to a speed well below the maximum of 1000 revolutions per minute. Due to anticipated difficulties in the turning of a shaft smaller than 0.3 inch diameter, this value was selected for the new test shaft. The computations for the theoretical natural frequency of the system with a test shaft of 0.3 inch diameter are shown in Appendix I. The speed at the natural frequency was found to be 840 revolutions per minute.

It was recognized that with the nodal point so close to the gear end of the test shaft, the angular oscillation of the pick-up gear would necessarily be small. It was therefore attempted at this point to check the natural frequency, realizing that the only hope for obtaining measurable oscillations of the pick-up gear lay in operating exactly at the natural fre-

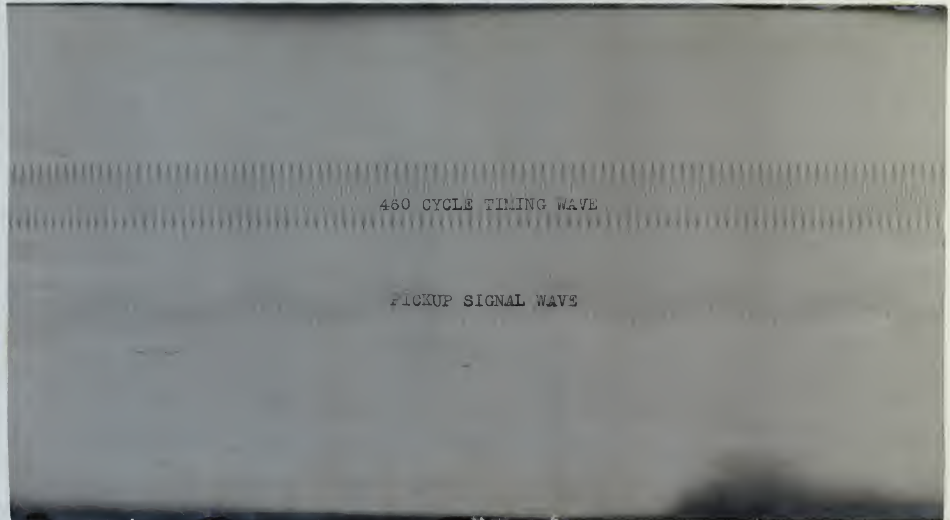

FIGURE I. Westinghouse Oscillogram; 900 Revolutions per Minute

quency of the system. Oscillograms were taken at the calculated natural frequency and at several speeds above and below this value but with negative results. The signal wave showed almost perfect uniformity, except for slight variations due to fluctuation of the basic speed of the driving unit or to fluctuations in film speed. The film speed fluctuation was detectable by reference to the constant frequency timing wave which was imposed upon all the oscillograms.

The speed was then carried to the 1000 revolutions per minute maximum and a series of continuous oscillograms were made as the speed was reduced gradually to 600 revolutions per minute. It was hoped that as the system passed through its natural frequency, a distinct and noticeable change in the signal wave lengths would appear. This method also gave negative results, the signal wave again remaining uniform, as before.

At this point it was decided that perhaps the wave lengths of the signal wave on the time base provided by the Westinghouse camera attachment were too small to permit proper indication of the expectedly minute oscillations of the pick-up gear. The Westinghouse instrument provided a maximum film speed of 3 feet per second, resulting in a wave length varying from 0.047 inch at 1000 revolutions per minute to 0.078 inch at 600 revolutions per minute. Therefore a General Electric oscillograph, having a maximum film speed of 5 feet per second was obtained.

Continuous oscillograms, similar to the ones taken with the Westinghouse oscillograph, were taken with the General

Electric instrument. The resulting wave lengths were easier to handle than were the Westinghouse wave lengths, but the indications of cyclic oscillation at a natural frequency were again negative.

With available time growing short, it was finally decided to add a flywheel at the driving end in order to move the nodal point farther from the gear end of the test shaft. The latter part of Appendix I contains the computations for the new natural frequency and nodal point for the system. It should be noted that many approximations were used in these computations, such as assuming the total moment of inertia of the driving rotor and flywheel to be acting at one point and disregarding the heavier sections of the shaft. The theoretical natural frequency of the modified system was found to be 489 revolutions per minute .

Continuous oscillograms were again run, this time from 600 revolutions per minute to 300 revolutions per minute. The preliminary investigation of the waves consisted of measurement of 23 cycle (half revolution) sections of the signal wave, searching for cyclic variations in their lengths. Several such cyclic variations were found to exist in the vicinity of 490-500 revolutions per minute. The region of these cyclic variations were then further investigated to establish acceleration and deceleration half revolutions and maximum and minimum angular velocities. The method of obtaining these various elements, as well as the method of using them in the analysis of the torsional-vibration of the system, are illustrated in Appendix II.

Due to lack of time, it was found to be impossible to carry out further refinements to the system which should give better results. These possible refinements are discussed in the following Chapter.

CHAPTER V

CONCLUSIONS

The results obtained from this investigation, although basic, definitely indicate that an oscillatory torque can be superimposed upon the rotation of a shaft by use of a device similar in character to the unit described in this paper. It is fully realized that the apparatus is subject to many major and minor improvements and modifications, which, it is believed, will result in more accurate and reliable data.

The pick-up gear and magnet arrangement yielded highly satisfactory results. However, it is believed that, from the standpoint of ease and accuracy of analysis, a better signal wave would be obtained by using a higher film speed on the oscillograph. This would give a longer wave length and allow for magnification of any variations in the wave length caused by torsional vibration. It would also be highly desirable to have a camera attachment which would give an accurately controlled film speed in order to eliminate wave length variations caused by variation of the film speed.

As has been indicated previously, the flywheel was added to improve the frequency characteristics of the system and also to move the nodal point of the shaft farther from the gear end. It is now considered that it would be highly advisable to replace the flywheel at the driving end with a pick-up gear and magnet identical with the pick-up gear and magnet at the driven end. The signals from the two magnets could then be introduced into two separate elements of a multi-element oscillograph, thus presenting a direct comparison between the behavior of the two ends of the equivalent shaft under test consideration without

reference to a theoretical nodal point.

Difficulty was experienced with speed control on the driving end during the running of constant speed tests. This was partially due to variations in the air supply caused by a long run of relatively small diameter pipe between the compressor and the inducer unit. Obviously, the remedy for this situation is to locate the unit close to the air supply and to provide adequate piping. Some thought might be given in the future to the use of a constant speed electric motor to drive the unit, with boosting and bucking air jets supplied by a separate unit, similar in construction to the present combined unit, which provides the driving torque as well as the boosting and bucking air jets.

It is considered that this investigation has laid the ground work for future investigations into the torsional-vibration of rotating shafts, using air jets to induce the torsional-vibration and an arrangement consisting of a pick-up gear and magnet, and an oscillograph to provide data for the computation of angular velocities, angular displacements, angular accelerations, torques, and stresses in the rotating shafts.

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APPENDIX I

THEORETICAL ANALYSIS

1. Natural Frequency of Torsional-Vibration

The simplest case of torsional-vibration is that of the simple torsional pendulum. In this arrangement a circular disk is attached to a shaft and the end of the shaft opposite the disk is fixed rigidly in a support. By twisting the shaft, torsional-vibrations of the system can be produced. The period of torsional-vibration will be given by the following equation:

$$T = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{I}{k}} = 2\pi\sqrt{\frac{Il}{GI_p}} \qquad (1)$$

In the case of a system consisting of two disks joined together by a shaft, the disks will always rotate in opposite directions during vibration and there will be a nodal section which will remain stationary during oscillation. The distances l_1 and l_2 of this section from the disks will be inversely proportional to the moments of inertia of the disks. Therefore, the values of l_1 and l_2 may be expressed as follows:

$$l_1 = \frac{l I_2}{(I_1 + I_2)} \quad ; \quad l_2 = \frac{l I_1}{(I_1 + I_2)} \qquad (2)$$

By substituting l_1 and I_1 in place of l and I in equation (1), the following equations for the period and frequency of a two disk system are obtained:

$$T = 2\pi\sqrt{\frac{I_1 I_2 l}{(I_1 + I_2) G I_p}} \qquad (3)$$

$$f = \frac{1}{2\pi}\sqrt{\frac{(I_1 + I_2) G I_p}{I_1 I_2 l}} \qquad (4)$$

In the case of a shaft of variable cross section, an "equivalent shaft" of constant cross section may be used. This conversion may be accomplished by use of the fact that to obtain the same angle of twist from two shafts of different cross sections, their lengths must vary directly as the fourth powers of their diameters. In equation form

$$l_{eq} = \frac{l d_{eq}^4}{d^4} \tag{5}$$

Based on the above equations the natural frequency of the system used in connection with this project may be computed as shown in the following paragraphs.

(a) Moment of Inertia of Driving Rotor.

The driving rotor was fabricated from one piece of steel. Its cross section consists of a heavy rim with brass blades set into milled axial slots, a thin web, and a long hub bored axially for the shaft. For the purpose of calculating the moment of inertia of the rotor, the following portions were treated separately:

(1) Rim; in the shape of a hollow cylinder,

5-3/8" O.D. x 4-1/8" I.D. x 3" long.

$$(I)_{rim} = \frac{M}{2}(R^2 + r^2) = \frac{0.02035\,(\overline{5.375}^2 + \overline{4.125}^2)}{2 \times 4}$$

$$= 0.1163 \text{ lb inches}$$

(2) Web; in the shape of a flat disk,

4-1/8" dia. x 1/4" thick.

$$(I)_{web} = \frac{Mr^2}{2} = \frac{0.00242 \times \overline{4.125}^2}{2 \times 4}$$

$$= 0.005160 \text{ lb inches}$$

(3) Hub; including the shaft within the hub; in the form of a solid cylinder, 1-13/16" dia. x 3-15/16" long.

$$(I)_{hub} = \frac{Mr^2}{2} = \frac{0.00735 \times \overline{1.813}^2}{2 \times 4}$$

$$= 0.00302 \text{ lb inches}$$

(4) Blades; 1-3/4" long x 17/32" high x 1/8" thick brass, with centroidal radius of 2-61/64".

$$(I)_{blades} = \frac{M}{12}(a^2 + b^2) + Mr_c^2$$

$$= \frac{0.00226\,(\overline{0.531}^2 + \overline{0.125}^2)}{12} + (0.00226 \times \overline{2.954}^2)$$

$$= 0.02022 \text{ lb inches}$$

(5) Total (I) of driving rotor

$$(I_2) = 0.1163 + 0.00516 + 0.00302 + 0.02022$$

$$= 0.14470 \text{ lb inches}$$

(b) Moment of Inertia of Pick-up Gear.

The pick-up gear is a 46-tooth deep spur gear of approximately 11" diameter, with a rim which is 1" thick, a web which is 7/32" thick and is lightened by 12-11/16" diameter holes, and a separate mounting hub which has an outside diameter of 2-7/8", an overall length of 3", and a mounting flange which has a diameter of 4-45/64" and a thickness of 1/4". For the purpose of calculating the moment of inertia of the gear, the hub was considered as solid and the gear was reduced into basic

shapes as follows:

(1) Web; in the shape of a flat disk, 10-3/4" dia. x 7/32" thick.

$$(I)_{web} = \frac{Mr^2}{2} = \frac{0.0144 \times \overline{10.75}^2}{2 \times 4}$$

$$= 0.208 \text{ lb inches}$$

(2) Rim; in the shape of a circular ring of rectangular cross section, 10-3/4" O.D. x 9-1/8" I.D. x 25/32" thk.

$$(I)_{rim} = \frac{M}{2}\left(R^2 + r^2\right) = \frac{0.0145\left(\overline{10.75}^2 + \overline{9.125}^2\right)}{2 \times 4}$$

$$= 0.358 \text{ lb inches}$$

(3) Mounting Flange; in the shape of a flat disk, 4-3/4" dia. x 1/16" thick.

$$(I)_{mf} = \frac{Mr^2}{2} = \frac{0.000805 \times \overline{4.75}^2}{2 \times 4}$$

$$= 0.00228 \text{ lb inches}$$

(4) Large Hub Diameter; in the shape of a cylinder, 2-7/8" dia. x 31/32" thick.

$$(I)_{hub} = \frac{Mr^2}{2} = \frac{0.00455 \times \overline{2.875}^2}{2 \times 4}$$

$$= 0.00470 \text{ lb inches}$$

(5) Small Hub Diameter; in the shape of a cylinder, 2" dia. x 1-3/4" thick.

$$(I)_{hub} = \frac{Mr^2}{2} = \frac{0.00399 \times \overline{2.0}^2}{2 \times 4}$$

$$= 0.00199 \text{ lb inches}$$

(6) Holes; in the form of 12 thin disks, of 11/16" dia. x 7/32" thick, and a centroidal radius of 3-11/16".

$$(I)_{holes} = 12\left[\frac{Mr^2}{2} + Md^2\right] = 0.00072\left[\frac{0.688^2}{2 \times 4} + 3.688^2\right]$$

$$= \quad 0.00984 \text{ lb inches}$$

(7) Total (I) of Pickup Gear

$$(I_1) = 0.20800 + 0.35800 + 0.00228 + 0.00470$$
$$+ 0.00199 - 0.00984 = 0.56513 \text{ lb inches}$$

(c) Length of Equivalent Shaft.

By use of equation (5) above, the various sections of the shaft which connects the two rotating masses may be reduced to an "equivalent shaft" of a uniform diameter. Considering only the sections of shaft between the inner ends of the hubs and referring all cross sections to a shaft with a diameter of 0.3", the following results are obtained:

(1) 1.344" dia. x 3.25" long.

$$(l_{eq})_a = \frac{3.25 \times 0.3^4}{1.344^4} = 0.00805\text{" long}$$

(2) 1.1814" dia. x 2.5" long.

$$(l_{eq})_b = \frac{2.5 \times 0.3^4}{1.1814^4} = 0.01036\text{" long}$$

(3) 0.75" dia. x 7.813" long.

$$(l_{eq})_c = \frac{7.813 \times 0.3^4}{0.75^4} = 0.200\text{" long}$$

(4) 0.30" dia. x 10.0" long.

$$(l_{eq})_d = \frac{10.00 \times 0.3^4}{0.3^4} = 10.0\text{" long}$$

(5) 1.00" dia. x 2.813" long.

$$(l_{eq})_e = \frac{2.813 \times 0.3^4}{1} = 0.02278\text{" long}$$

(6) 1.312" dia. x 1.906" long.

$$(l_{eq})_f = \frac{1.906 \times 0.3^4}{1.312^4} = 0.00520\text{" long}$$

(7) Total Equivalent Length for 0.30" dia. shaft.

$$l_{eq} = 0.00805 + 0.01036 + 0.200 + 10.00 + 0.02278 + 0.00520 = 10.24639\text{" long.}$$

(d) Location of Nodal section.

By substitution in equations (2), the equivalent distances from the gear end to the nodal section may be computed.

$$(l_1) = \frac{l_{eq} \times I_2}{I_1 + I_2} = \frac{10.247 \times 0.1447}{0.1447 + 0.5651}$$

$$= 2.0888\text{" from gear end.}$$

Referring this value back to the true sizes of the shaft cross sections will place the nodal section at a distance of 1.9959" from the gear end of the 0.30" diameter section of the test shaft.

(e) Natural Frequency of the System.

By substitution in equation (4), the natural frequency of torsional-vibration for the system may be computed.

$$f = \frac{1}{2\pi}\sqrt{\frac{(I_1 + I_2) G I_p}{I_1 \times I_2 \times l_{eq}}} = \frac{1}{2\pi}\sqrt{\frac{6489.62}{0.8379}}$$

= 14 cycles per second

or 14 x 60 = 840 revolutions per minute

2. Approximate Value of Natural Frequency of Torsional-Vibration of System with Flywheel Added to Driving End

For the purpose of approximation the system consisting of the driving rotor and flywheel at one end and the pick-up gear at the other end was assumed to be a two mass system. Only the reduced cross section (0.30") shaft was considered as driving the gear. The 9 pound weight of the flywheel was

Prob D2.

(a)

Energy per unit mass $= \dfrac{40 \times 144}{\rho g} = 89.6$ ft (head)

Energy at B $= 89.6$ ft-lbs/lbs.

Energy at D = Energy at B − Losses.

$V_D = \frac{8}{9} V_B$ $\qquad$ Losses $= \dfrac{V_C^2}{2g} = \dfrac{V_B^2}{16g}$ ft-lbs/lb

$V_C = \dfrac{V_B}{2}$

$$89.6 = \frac{V_D^2}{2g} - \frac{V_B^2}{16g} = \frac{64}{81}\frac{V_B^2}{2g} + \frac{V_B^2}{\cancel{16}\, 8g} = \frac{40 \times 144}{2g}$$

$$.915\ V_B^2 = 5760$$

$$V_B^2 = 6300$$

$$V_B = 79.3 \text{ ft/sec.}; \quad V_D = 70.5 \text{ ft/sec}$$

$$m_s = V_B A_B \rho g = 79.3 \times 64.4 \times \frac{\pi}{4} \times \frac{1}{16} \times \frac{1}{144}$$

$$m_s = \cancel{1.602}\ 1.745 \text{ lbs/sec.}$$

(b) Forces + moments to support [illegible]

$$\frac{40 \times 144}{2g} = \frac{V_B^2}{2g} + \frac{p_B}{\rho g}$$

$$p_B = 40 \times 144 \quad V_B^2 = 5760 - 6300$$

$$p_B = -540 \text{ psfg} = -3.75 \text{ psig}$$

$$R = +i_3\left[(79.3)^2 \times 2 \times 3.41 \times 10^{-4}) - (540 \times 3.41 \times 10^{-4}) + 2\left(\sin 30^\circ \times 70.5^2 \times 2 \times \frac{9\pi}{256} \times \frac{1}{4} \times \frac{1}{144}\right)\right]$$

$$= i_3\left[4.11 + 1.885\right] = 5.995 \text{ lbs}$$

$$\bar{m} = -i_3\left[\cos 30^\circ \times 70.5^2 \times 2 \times \frac{9\pi}{256} \times \frac{1}{4} \times \frac{1}{144} \times \frac{2}{6}\right]$$

$$\bar{m} = -i_3\left[0.544\right] \text{ ft-lbs}$$

.79
.125
.865
.915

Parent I.

$$\frac{V_1^2}{2g} + \frac{p_1}{\rho g} + z_1 = \frac{V_2^2}{2g}$$

$$\frac{10^4}{2g} + \frac{p}{2g} + 12 = \frac{4\times10^4}{2g}$$

$$p_1 = 29227 \; psfg = 203 \; psig$$

$$G_1 = G_2 = V_1\rho = V_2\rho = 100\times2 = 200 \text{ slugs}/ft^2/sec.$$

$$\bar{R}_1 = \int \bar{V}\, G\, ds' + \int \bar{n}\, p\, ds' = -i_3\left[\left(100\times200\times\tfrac{\pi}{4}\right) + 29227\times\tfrac{\pi}{4}\right]$$

$$\bar{R}_1 = -i_3\,(38{,}700) \text{ lbs.}$$

$$\ddot{R}_1 = R_1 = -i_3\,(38{,}700)$$

$$M_1 = 0 \;;\; M_2 = i_2(10)(38{,}700) = i_2(387{,}000) \text{ ft lbs.}$$

$$G_3 = 2G_1 = 400 \text{ slugs}/ft^2/sec$$

$$\bar{R}_3 = -i_1\left(\tfrac{20}{\sqrt{425}}\right)\left[\left(200\times400\times\tfrac{\pi}{4}\right)\right] + i_3\left(\tfrac{5}{\sqrt{425}}\right)\left(200\times400\times\tfrac{\pi}{4}\right)$$

$$R_3 = -i_1\,(6.10)\times10^4 \qquad + i_3\,(1.525)\times10^4$$

$$M_3 = 0$$

Force on "A"

$$\ddot{R}_A = -i_1(6.1)\times10^4 - i_2(6.215)\times10^4 \;;\; |R_A| = 8.7\times10^4 \text{ lbs.}$$

$$M_A = i_2(387{,}000) \text{ ft lbs} = i_2(38.7)\times10^4 \;;\; |M_A| = 38.7\times10^4 \text{ lbs} \text{ clockwise}$$

considered to be concentrated in the rim.

Then the moment of inertia of the flywheel, considered as a circular ring is

$$I_3 = \frac{M}{2}(R^2 + r^2) = \frac{9}{2 \times 386}(36 + 25)$$

$$= 0.771 \text{ lb inches}$$

The moment of inertia of the driving end is then

$$I_2 + I_3 = 0.1447 + 0.7710 = 0.9157 \text{ lb inches}$$

With the moment of inertia of the pick-up gear remaining as before, the natural frequency of the system will then be

$$f = \frac{1}{2\pi}\sqrt{\frac{(0.91570 + 0.56513) \times 11.5 \times 10^6 \times 0.000795}{0.91570 \times 0.56513 \times 10}}$$

f = 8.15 cycles per second

or 8.15 x 60 = 489 revolutions per minute.

3. Nodal Section for System with Flywheel added.

$$l_1 = \frac{l_{eq}(I_2 + I_3)}{I_1 + I_2 + I_3} = \frac{10 \times 0.91570}{0.91570 + 0.56513}$$

= 6.19" from gear end of test shaft.

APPENDIX II

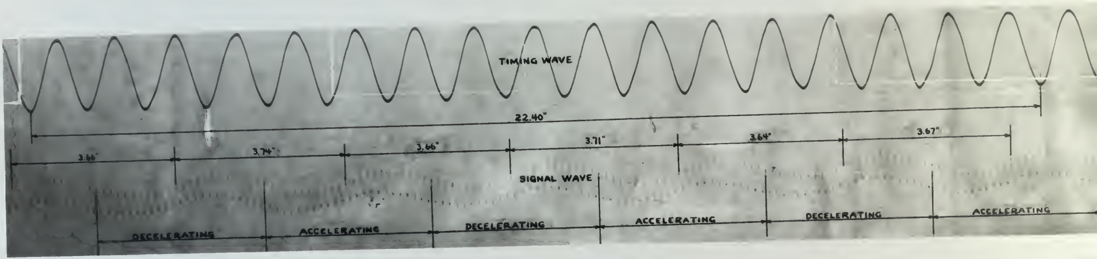

TIMING WAVE
22.40"
3.66"
3.74"
3.66"
3.71"
3.64"
3.67"
SIGNAL WAVE
DECELERATING
ACCELERATING
DECELERATING
ACCELERATING
DECELERATING
ACCELERATING

FIGURE II

ANALYSIS OF RESULTS

For the case represented by Figure II the cyclic frequency variation indicating torsional-vibration persisted during three full revolutions of the pick-up gear. The lengths of the half-cycles were measured and their lengths have been indicated on the Figure. For the purpose of this analysis it was decided to average the three maximum half-cycles, the three minimum half-cycles, and the three full cycles. These averages were as follows:

Average maximum half-cycle	3.66 inches
Average minimum half-cycle	3.71 inches
Average full cycle	7.36 inches

The timing wave which was imposed upon the oscillogram was of a frequency of 46 cycles per second. This frequency was selected because there were 46 teeth in the pick-up gear so that a signal wave with a frequency of 46 cycles per second would represent a speed of 1 revolution per second or 60 revolutions per minute. The timing wave and signal wave were measured over the total length of the three cycles with the following results:

Timing wave - 17 cycles - 22.40 inches

Signal wave - 138 cycles - 22.09 inches

Then, by means of a simple proportion, the average speed of the pick-up gear during the period under study was:

$$\frac{22.4}{17} \times \frac{138.0}{22.09} \times 60 = 494 \text{ RPM}.$$

The half cycle spanning the point of maximum velocity

represented 23 cycles occurring in a space of 3.66 inches. Thus the average speed of the pick-up gear during the accelerating half-cycle was:

$$\frac{22.4}{17} \times \frac{23}{3.66} \times 60 = 497 \text{ RPM.}$$

Similarly, the half-cycle spanning the point of minimum velocity was found to represent 23 cycles in a space of 3.71 inches. Thus the average speed of the pick-up gear during the decelerating half-cycle was:

$$\frac{22.4}{17} \times \frac{23}{3.71} \times 60 = 490 \text{ RPM.}$$

Converted into terms representing angular velocities, the above speeds are:

Basic:

$$\omega = \frac{494}{60} \times 2\pi = 51.70 \text{ rad/sec.}$$

Maximum:

$$\omega_{max} = \frac{497}{60} \times 2\pi = 52.02 \text{ rad/sec.}$$

Minimum:

$$\omega_{min} = \frac{490}{60} \times 2\pi = 51.29 \text{ rad/sec.}$$

It appeared to be impracticable with the equipment at hand to accurately measure the individual wave lengths which would yield instantaneous angular velocities. It was therefore decided to use, for the purpose of this approximate analysis, the average angular velocities over the half-cycles spanning the maximum and minimum velocities to represent, respectively, the actual maximum and minimum angular velocities of the pick-up gear. It was further assumed that the pick-up gear was oscillating with simple harmonic motion in torsion between these

extremes of angular velocity. In line with this assumption, the mean angular velocity, or that constant angular velocity with which the nodal point of the shaft will be rotating, will be:

$$\omega_{mean} = \frac{51.29 + 52.02}{2} = 51.65 \text{ rad/sec.}$$

Then, the magnitudes of the angular velocities of the pick-up gear may be represented by a sinusoidal wave having a maximum amplitude of (51.65 + 0.366) radians per second and a minimum amplitude of (51.65 - 0.366) radians per second. In equation form this curve may be represented as

$$\omega = \omega_n + 0.366 \cos \omega_n t$$

where $\omega_n = 51.65$ and t = time (sec).

Then

$$\omega = 51.65 + 0.366 \cos(51.65t)$$

The angular acceleration of the pick-up gear will be represented by the first derivative with respect to time of the above expression. Or:

$$\alpha = \frac{d\omega}{dt} = -(0.366 \times 51.65t) \sin(51.65t)$$

The maximum acceleration will occur where sin (51.65t) = -1 or where $(51.65t) = \frac{3\pi}{2}$ radians. Likewise, the maximum deceleration (which is merely negative acceleration) will occur where sin (51.65t) = +1, or where $(51.65t) = \frac{\pi}{2}$ radians. Therefore:

$$\alpha_{max.} = 0.366 \times 51.65 \times (\pm 1) = \pm 18.90 \text{ rad/sec}^2$$

Wilson (7) develops an equation involving the angular

This value is well within the tabulated value for the elastic limit of cold rolled steel.

acceleration, amplitude of angular oscillation, phase velocity of vibration, and time. This equation, using the notation adopted for this paper, is given as follows:

$$\alpha = -\omega_n^2 a \sin \omega_n t$$

By transposing the above equation and substituting known values, the amplitude of vibration may be computed

$$a = -\frac{\alpha}{\omega_n^2 \sin \omega_n t}$$

Since $\alpha = +18.90$ where $\omega_n t = \frac{3\pi}{4}$, the maximum value of a will be

$$a = -\frac{18.90}{51.65 \times (-1)} = 0.00708 \text{ radians.}$$

Since this represents the angular displacement from the mean position, the total angle of twist of the shaft will be

$$\theta = 2a = 0.01416 \text{ radians}$$

$$\text{or } 2 \times 0.01416 = 0.81 \text{ degrees.}$$

The resultant maximum stress in the test shaft may be computed by using the following equations. with substitutions as indicated:

$$\text{Torque} - T = \frac{\theta G I_p}{l} = \frac{0.01416 \times 11.5 \times 10^6 \times 0.000795}{6.19}$$

$$= 20.92 \text{ pound inches.}$$

Maximum stress

$$S_s = \frac{16T}{\pi d^3} = \frac{16 \times 20.92}{3.14 \times 0.3^3}$$

$$= 3945 \text{ pound inches.}$$

APPENDIX III

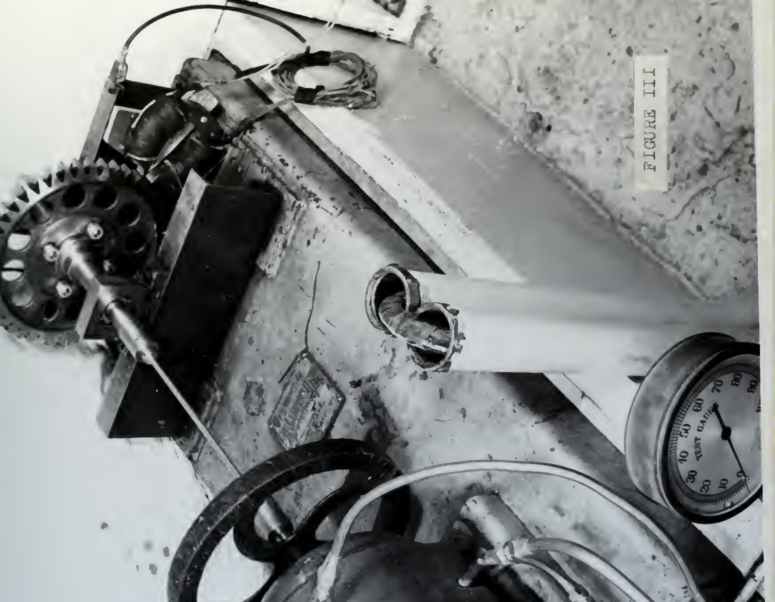
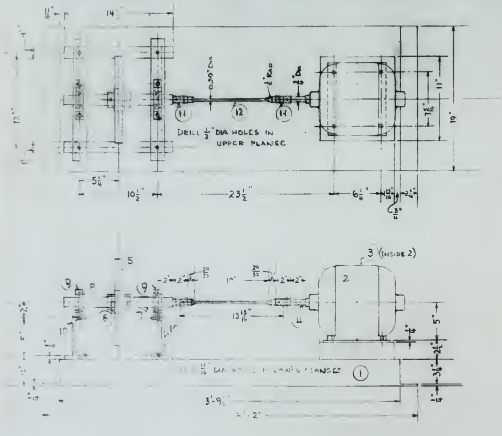


FIGURE III

FIGURE IV

FIGURE V



... Vibration Inducer

Scale 2' = 1'

	Bill of Material		
Part No.	Description	No Req'd	Mat'l
1	Mounting Base (Furnished)	1	Steel
2	Air Motor Stator (Motor Frame Furn)	1	"
3	Air Motor Rotor	1	Steel & Brass
4	Rotor Shaft	1	Steel
5	Pickup Gear	1	"
6	Gear Hub	1	"
7	Gear Shaft	1	"
8	No. 206R Ball Bearings (Furn)	2	
9	Bearing Brackets	2	Steel
10	5" Channel Iron, 16" Long	2	"
11	Sleeve Couplings	2	"
12	Connecting Shaft	1	"

Notes

1. Material for fabricated parts to be supplied by E.E.S.
2. Flange faces of 5" channels to be made parallel
3. Sleeve couplings to be fabricated from 1" double extra heavy pipe bored to $\frac{3}{4}$" I.D and assembled to shafts with light drive fit, secured with taper pins
4. Assembly to be by P.G. School Personnel.

U.S. Naval Postgraduate School
Torsional Vibration Inducer
Assembly

FIGURE VI

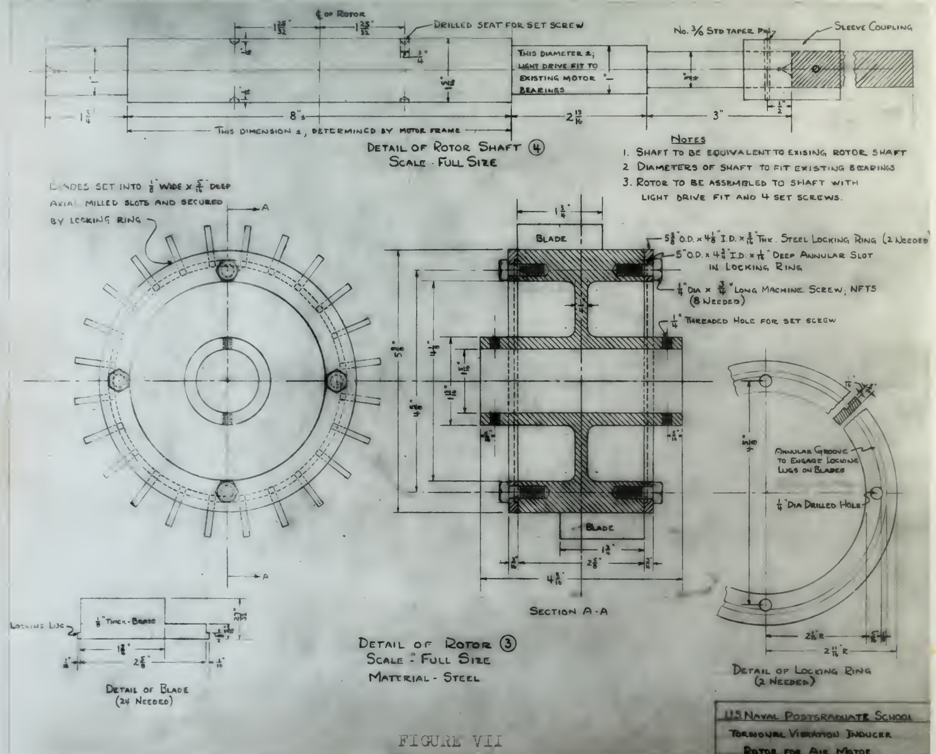
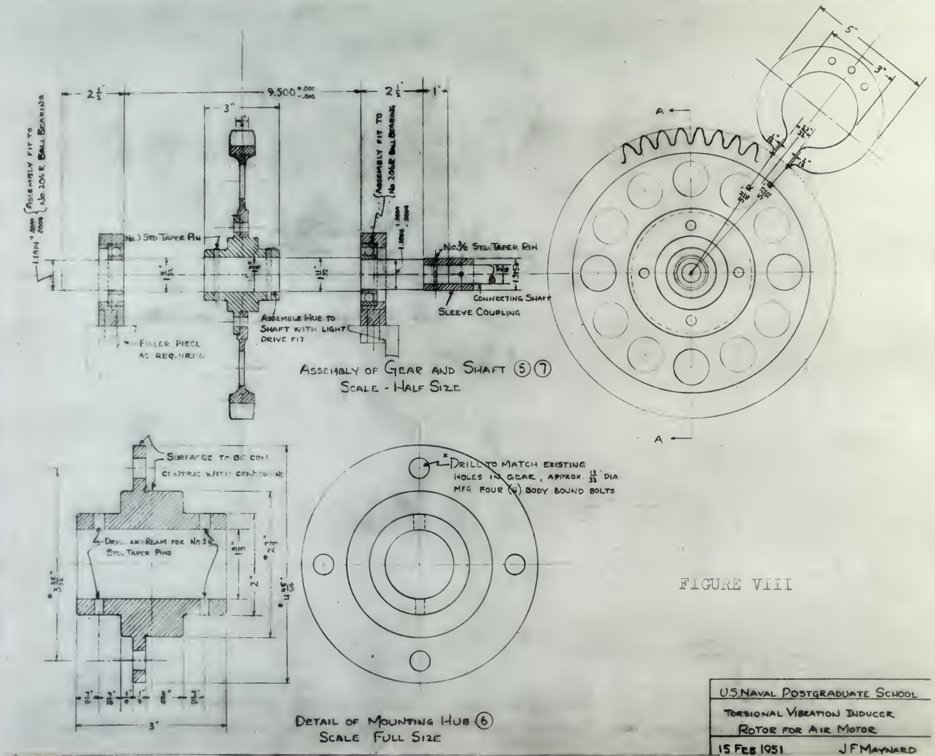

DRILLED SEAT FOR SET SCREW
No. 3/0 STD TAPER PIN
SLEEVE COUPLING
THIS DIAMETER ±; LIGHT DRIVE FIT TO EXISTING MOTOR BEARINGS
THIS DIMENSION ±, DETERMINED BY MOTOR FRAME
DETAIL OF ROTOR SHAFT (4)
SCALE - FULL SIZE
NOTES
1. SHAFT TO BE EQUIVALENT TO EXISING ROTOR SHAFT
2 DIAMETERS OF SHAFT TO FIT EXISTING BEARINGS
3. ROTOR TO BE ASSEMBLED TO SHAFT WITH LIGHT DRIVE FIT AND 4 SET SCREWS.
BLADES SET INTO 1/8" WIDE x 5/16" DEEP AXIAL MILLED SLOTS AND SECURED BY LOCKING RING
BLADE
5 5/8" O.D. x 4 1/8" I.D. x 3/16" THK. STEEL LOCKING RING (2 NEEDED)
5" O.D. x 4 3/4" I.D. x 1/16" DEEP ANNULAR SLOT IN LOCKING RING
1/4" DIA x 3/4" LONG MACHINE SCREW, NFTS (8 NEEDED)
1/4" THREADED HOLE FOR SET SCEW
SECTION A-A
ANNULAR GROOVE TO ENGAGE LOCKING LUGS ON BLADES
1/4" DIA DRILLED HOLE
DETAIL OF LOCKING RING (2 NEEDED)
LOCKING LUG
1/8" THICK - BRASS
DETAIL OF BLADE (24 NEEDED)
DETAIL OF ROTOR (3)
SCALE : FULL SIZE
MATERIAL - STEEL
U.S. NAVAL POSTGRADUATE SCHOOL
TORSIONAL VIBRATION INDUCER
ROTOR FOR AIR MOTOR

FIGURE VII



2 1/2"
9.500 +.000 -.010
2 1/2"
1"
3"
Assembly fit to No. 206 R Ball Bearing
Assembly fit to No. 206 R Ball Bearing
No. 3 Std Taper Pin
No. 3/0 Std. Taper Pin
Connecting Shaft
Sleeve Coupling
Assemble Hub to Shaft with light drive fit
Filler piece as required
Assembly of Gear and Shaft (5) (7)
Scale - Half Size
A
A
5"
3"
Surfaces to be concentric with centerline
Drill and Ream for No 3 Std. Taper Pins
2"
3"
Drill to match existing holes in gear, approx. 13/32" dia
MFG four (4) body bound bolts
Detail of Mounting Hub (6)
Scale Full Size
U.S. Naval Postgraduate School
Torsional Vibration Inducer
Rotor for Air Motor
15 Feb 1951
J F Maynard

FIGURE VIII